Engineered shock and vibration solutions

Vibro/Dynamics and the Socitec Group have specialized in shock and vibration solutions for more than 50 years. In 2014, Vibro/Dynamics became part of the Socitec Group. This merger has produced an organization with offices in the US, Europe, and Asia to support a diverse, global customer base of installations in more than 80 countries. As a US company headquartered in the Chicagoland area, Vibro/Dynamics can utilize the combined engineering expertise to offer advanced, reliable solutions tested and approved by some of the world's largest and most respected companies.

Our products and services include:

- Unrivaled range of Elastomeric and Wire Rope Isolators (WRIs) for shock and vibration isolation of military and industrial systems including standardized commercial off-the-shelf (COTS) equipment
- Custom engineered isolation solutions manufactured to meet individual requirements
- Data Loggers that record location, time, acceleration, temperature, relative humidity, inclination, air pressure and more
- Computer simulations with proprietary software
- On-site testing

Shock and vibration design and consultancy

The specialized product range is supported by the following capabilities:

(a) Comprehensive advice and design support

Our vast experience working in all types of shock and vibration applications allows us to provide a complete engineering service including:

- Improving product quality and reliability
- Determining critical design features before going to test
- Optimizing component mass and rigidity

(b) Advanced simulation techniques

- Nonlinear one degree of freedom (1-DOF) systems
  SDNL1, a proprietary Socitec Group program, as part of our complimentary engineering support, enables us to predict the response of simple (uncoupled) systems. SDNL1 utilizes a database of over 10,000 cases, allowing us to quickly relate new applications with previous experience.
- Nonlinear multiple degree of freedom systems
  SYMOS, an in-house developed n-DOF simulation software package, allows us to simulate complex systems involving nonlinearities (up to 500-DOF), in order to predict shock and vibration isolation performance.
- Finite Element Analysis (FEA)
  The use of FEA programs creates a very detailed picture of dynamic and structural loading, fine-tuned to meet the most demanding shock and vibration requirements.

(c) Broad range of physical testing

Our specialized environmental test capabilities include:

- Shock and vibration measurement
- Collaboration with customers during qualification testing
- Temperature cycling -150°F to +400°F

Our broad range of testing capabilities, numerous resources, and expertise in simulation software allow us to provide services that will reduce prototyping and product development costs. This catalog encompasses most, but not all of Vibro/Dynamics' expertise in solving shock and vibration problems. Please call us to discuss your application, and our team of experienced engineers will work closely with you to find the optimal solution. There is hardly any application that we have not encountered.

Fundamentals of shock and vibration

1. Simple case: 1-DOF, mass-spring-damper model

Analysis of simple shock and vibration problems is based on the 1-DOF harmonic oscillator depicted in Figure 1, and is generally referred to as the mass-spring-damper model. The forces acting in the system and the motion behavior are governed by three components: 'm' represents mass, 'k' represents the stiffness component (depicted as a spring), which exerts a restoring force proportional to its deformation 'x', and finally a dashpot 'c' which supplies a force proportional to the relative velocity of motion in the spring, otherwise known as viscous damping. This simplified system, or a summation of these systems can be used to represent a range of rigid body dynamic behaviors.

The following analysis focuses on the specific case of base excitation; that is the input to the system is from the base displacement (u). This specific case was selected because isolation of base excitation is the most typical application encountered in the naval and transportation fields. That being said, Vibro/Dynamics has the expertise to find a solution for any type of shock and vibration, including those generated by internal forces.

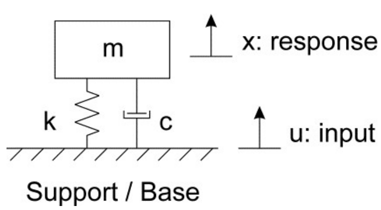

Figure 1. Mass spring damper model

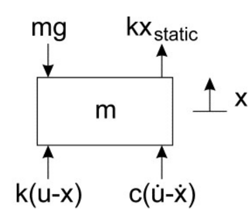

Figure 2. Free Body (FBD) of the sprung mass, m

Applying Newton's second law to the FBD shown in Figure 2, the equation of motion of the mass-spring-damper system can be derived as an ordinary differential equation:

$$\sum F = ma$$

$$kx_{static} - mg + c(\dot{u} - \dot{x}) + k(u - x) = m\ddot{x}$$

Since: $kx_{static} = mg$, then these two expressions cancel each other out.
Putting the response dependent variables to the left side of the equation and the input dependent variables to the right side:

$$m\ddot{x} + c\dot{x} + kx = c\dot{u} + ku$$

Dividing by m:

$$\ddot{x}(t) + 2\omega_n\xi\dot{x}(t) + \omega_n^2 x(t) = 2\omega_n\xi\dot{u}(t) + \omega_n^2 u(t)$$

$\omega_n = \sqrt{\frac{k}{m}}$ is called the undamped natural frequency (rad/s), and $\xi = \frac{c}{2\sqrt{km}}$ is called the viscous damping ratio.

This is the general form of the ordinary differential equation of the mass-spring-damper system excited by base motion. Depending on the input, different methods are used for solving the differential equation of motion. When the input is low amplitude over a repeated number of cycles, the problem is categorized as a vibration problem, whereas it is a shock problem when the input is high amplitude with short duration. In practice, the difference between shock and vibration is unclear, and systems will experience both shock and vibration. Therefore knowing how to account for both is critical.

In the following shock and vibration analyses only the underdamped case (i.e. $0 < \xi < 1$) is considered, as all Vibro/Dynamics' isolators have damping ratios well below one.

2. Vibration theory

A system is said to experience forced vibration when it is subjected to steady-state excitations. Some examples of such systems are: a shipping container with sensitive equipment transported on a truck, a naval electronics cabinet on a piston or turbine engine driven boat, and a centrifuge with unbalanced weight near sensitive scales and electronics. In the case of the unbalanced centrifuge, isolators are used to reduce the level of vibration transmitted to the base and nearby sensitive equipment. In the cases of the container and cabinet, the purpose of the isolator is to reduce the vibrations within the sprung equipment from vibrations coming through the base.

Linear systems

2.1.1 Harmonic base excitation vibration

A commonly encountered type of excitation is the harmonic base excitation:

$$u(t) = u_0 \sin(\omega t)$$

u_0: displacement amplitude (in) of the base, $\omega = 2\pi f$: input driving frequency (rad/s).

Solving the ordinary differential equation, with $u(t) = u_0 \sin(\omega t)$, gives the following steady-state response:

$$x(t) = x_0 * \sin(\omega t - \varphi)$$

x_0: displacement amplitude of the sprung mass, φ: phase shift between the displacement of the base and of the sprung mass.

An important conclusion can be made from this result: the response of any linear mass-spring-damper system experiencing harmonic base excitation is also harmonic, oscillating at the same input forcing frequency.

2.1.2 Vibration isolation with harmonic base excitation

For a system experiencing harmonic base excitation, correct implementation of a vibration isolator to the system will attain a condition wherein the acceleration transmitted to the sprung mass, m, is less than the input acceleration of the base. The performance of an isolator is best evaluated through the transmissibility (T), which is the ratio of the maximum acceleration of the sprung mass ($\ddot{x}_0$) to the maximum acceleration of the base ($\ddot{u}_0$):

$$T = \frac{\ddot{x}_0}{\ddot{u}_0} = \sqrt{\frac{1 + \left(2\xi \frac{\omega}{\omega_n}\right)^2}{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\xi \frac{\omega}{\omega_n}\right)^2}} = \frac{F_T}{F_0}$$

$\omega_n = 2\pi f_n$: undamped natural frequency (rad/s), F_0: magnitude of the harmonic force input applied to the sprung mass, F_T: magnitude of the force transmitted to the base.

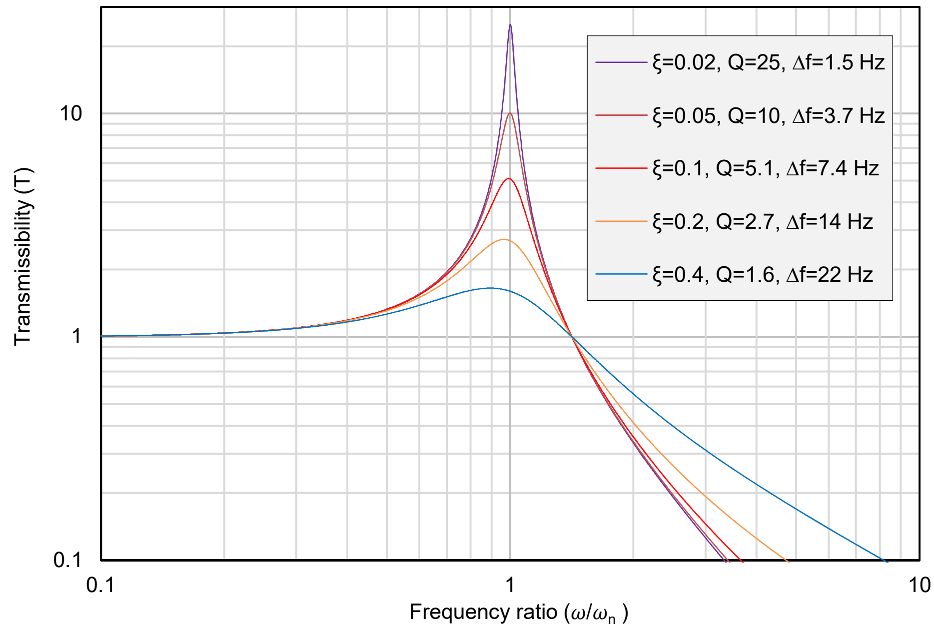

Figure 3. Typical transmissibility curves with varying damping ratios

The motion of the base and of the sprung mass may be expressed in terms of displacement, velocity, or acceleration, and the same transmissibility expression holds in each case. In this analysis, the motion is expressed in terms of acceleration because the acceleration experienced by the sprung mass is the most relevant factor in practice. Additionally, the transmissibility equation also holds for a mass-spring-damper system excited by a harmonic force: $F = F_0 \sin(\omega t)$.

The Q factor is commonly used in vibration theory. It is defined as the transmissibility at resonance, and is inversely proportional to the amount of damping in the system. Mathematically, the Q factor is defined as:

$$Q = \frac{f_r}{\Delta f} = \sqrt{\frac{1 + 4\xi^2}{4\xi^2}}$$

$$Q \approx \frac{1}{2\xi}$$ (for small damping ratio)

f_r: resonant frequency (Hz), Δf: frequency bandwidth (Hz).

The frequency bandwidth is the range of frequencies over which the transmissibility value is $(Q/\sqrt{2})$ and above. Therefore, for a given system with a fixed resonant frequency, the higher the Q factor the narrower the frequency bandwidth, as seen in Figure 3.

Looking at the transmissibility curves, three different regions can be established:

1. The *static region*: when ω/ω_n is much less than 1, the transmissibility approaches unity independently of the damping ratio. This is because at small frequency vibration the system's acceleration is very slow, and thus it responds statically by transmitting all of the base acceleration to the sprung mass.

2. The *resonance region*: when ω/ω_n approaches 1, the transmissibility is at a maximum; it is infinite for an undamped system. This is called resonance, and the major reason for vibration analysis is to predict when it may occur and take preventative steps. In this region, a higher damping ratio can significantly reduce the magnitude of the vibration and therefore reduce the transmissibility as well.

3. The *isolation region*: when ω/ω_n is much larger than 1, the transmissibility goes below 1 and the isolator works to reduce the acceleration transmitted from the base to the sprung mass. This is the region at which isolators are designed to be applied, since this is where they are most effective for vibration isolation. In this region, increasing the damping in the isolator actually reduces its isolation performance.

2.1.3 Random vibration

Base excitation with only one frequency is seldom encountered in practice, as most excitations are random; where different frequencies act simultaneously. The theory discussed above for a single frequency excitation still applies for linear systems under random vibration. Any random signal can be analyzed as the sum of many single frequency sinusoidal waves by means of a Fourier analysis. Therefore for linear systems, the response of each of those individual sinusoidal waves can be found and summed using superposition to find the global response of the system.

If the Fourier analysis is applied with a sufficiently fine frequency resolution, then the discrete frequency values can be considered as continuous and the concept of the power spectral density (PSD) can be developed. In order to use the PSD, the analyzed time history signal has to be ergodic and stationary as explained below:

1. The signal is *ergodic*; i.e. no matter the duration of the analyzed sample, its average values are always the same. This means that taking the average acceleration from a sample time history signal of duration Δt_1 and taking the average acceleration from a sample time history signal of duration Δt_2 yields the same result.

2. The signal is *stationary*; i.e. the PSD will be the same independent of the instant of time from where the sample is taken.

All in all, if the signal is ergodic and stationary then no matter at what instant of time the sample time history is taken and how long it is, the resulting PSD plot will be the same. An entire theory of analysis called spectral analysis has been developed around the PSD, and an example of its main use is shown below.

The PSD of the input signal in Figure 4 can be used to obtain the root mean squared (RMS) acceleration of that signal:

$$a_{rms,input} = \sqrt{\int_{f_1}^{f_2} PSD_{input}\, df}$$

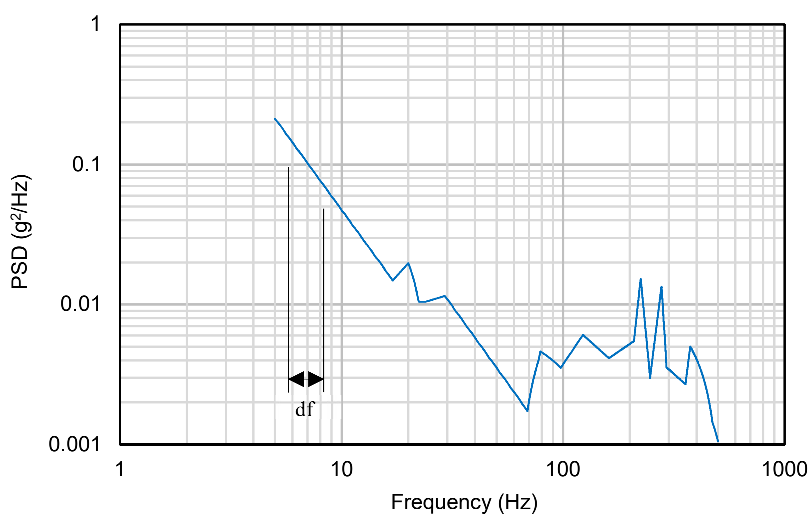

Figure 4. PSD of a typical random input signal seen on a vehicle

Usually in practice the response RMS acceleration is the one of interest, however the response PSD is rarely known. It is still possible in this case, to find the response RMS acceleration assuming a linear system.

For a linear system:

$$PSD_{response} = T^2 PSD_{input}$$

Where:

$$T = \frac{\ddot{x}_0}{\ddot{u}_0} = \sqrt{\frac{1 + \left(2\xi \frac{f}{f_n}\right)^2}{\left(1 - \frac{f^2}{f_n^2}\right)^2 + \left(2\xi \frac{f}{f_n}\right)^2}}$$

Then:

$$a_{rms,response} = \sqrt{\int_{f_1}^{f_2} T^2 PSD_{input} df} = \sqrt{\frac{2\pi f_n PSD_{input}}{8\xi}}$$

Through this equation, it is possible to determine the response RMS acceleration without knowing any information about the response time history signal or its PSD.

2.2 Nonlinear systems

Everything above applies only to linear systems, never seen in the real world. For nonlinear systems, the differential equation of motion is still valid, but not with constant coefficients—meaning the solution is no longer purely sinusoidal. A better way to represent the WRIs is by the elastically connected Coulomb damper model, and a better way to represent the elastomers is by the elastically connected viscous damper model, both of which are shown below.

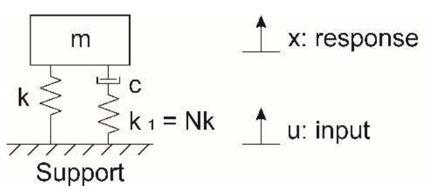

Figure 5. Elastically connected viscous damper

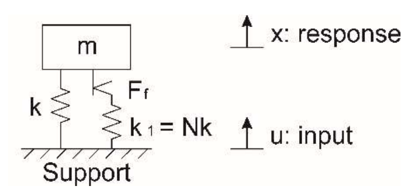

Figure 6. Elastically connected

The transmissibility equations for each of the two models are shown below, assuming a harmonic base excitation: $u(t) = u_0 \sin(\omega t)$.

Elastically connected viscous damper:

$$T = \frac{x_0}{u_0} = \sqrt{\frac{1 + 4\left(\frac{N+1}{N}\right)^2 \left(\xi \frac{\omega}{\omega_n}\right)^2}{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(\frac{2\xi\omega}{N\omega_n}\right)^2 \left(N + 1 - \frac{\omega^2}{\omega_n^2}\right)^2}}$$

This equation only holds for optimum damping, i.e. the value of damping which produces the minimum transmissibility at resonance, defined by:

$$\xi_{op} = \sqrt{2(N+2)}\, \frac{N}{4(N+1)}.$$

Elastically connected Coulomb damper:

$$T = \frac{x_0}{u_0} = \sqrt{\frac{1 + \left(\frac{4}{\pi}\mu\right)^2 \left[\left(\frac{N+2}{N}\right) - 2\left(\frac{N+1}{N}\right)\left(\frac{\omega_n}{\omega}\right)^2\right]}{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2}}$$

$\mu = F_f/(ku_0)$, is called the Coulomb damping parameter. The equation above applies only for N=3.

Such equations are not easily solved, so it is more practical in most cases to "linearize" them by using an equivalent mass-spring-damper model, as explained below.

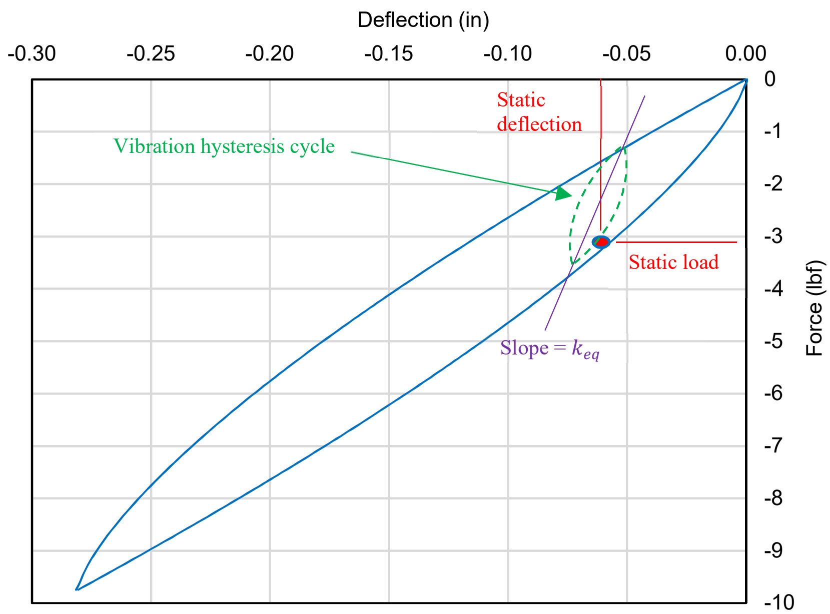

Figure 7. Hysteresis curve of a typical WRI in compression

The equivalent dynamic stiffness (k_{eq}) is the slope in the hysteresis cycle shown above.

In practice, there are many different forms of energy dissipation (friction damping, viscous damping, hysteretic damping, etc.), and not all of them lead to linear solutions. Only mass-spring-damper systems lead to linear solutions, which are easier to solve. This is why it is beneficial, for any vibrating system, to find its equivalent viscous damping ratio (ξ_{eq}) and its equivalent dynamic stiffness (k_{eq}), and then insert both into the simpler linear equations of a mass-spring-damper system:

$$T = \frac{x_0}{u_0} = \sqrt{\frac{1 + \left(2\xi_{eq}\frac{\omega}{\omega_n}\right)^2}{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\xi_{eq}\frac{\omega}{\omega_n}\right)^2}} = \frac{F_T}{F_0}$$

Since:

$$\omega_n = \sqrt{\frac{k_{eq}}{m}}$$

The transmissibility equation becomes:

$$T = \frac{x_0}{u_0} = \sqrt{\frac{1 + \left(2\xi_{eq}\omega\sqrt{\frac{m}{k_{eq}}}\right)^2}{\left(1 - \omega^2\frac{m}{k_{eq}}\right)^2 + \left(2\xi_{eq}\omega\sqrt{\frac{m}{k_{eq}}}\right)^2}} = \frac{F_T}{F_0}$$

The equivalent viscous damping is found by equating the area within the dashed ellipse in Figure 7, to the area in a viscously damped model vibrating at the same frequency and displacement amplitude. By doing this, the energy dissipated by the actual isolator is equated to the energy dissipated by an equivalent viscously damped isolator, per vibration cycle. The equivalent viscous damping is given by:

$$C_{eq} = \frac{E_d}{\pi\omega x_0^2}$$

E_d: dissipated energy per cycle, ω: vibration frequency in rad/s, x_0: amplitude of vibration

This equivalent viscous damping is then converted to an equivalent viscous damping ratio by:

$$\xi_{eq} = \frac{C_{eq}}{C_c} = \frac{C_{eq}}{2\sqrt{k_{eq}m}}$$

Plotting the equivalent dynamic stiffness (k_{eq}) and the equivalent viscous damping (C_{eq}) vs. isolator displacement amplitude with a 10 Hz frequency vibration would result in the following diagrams for a WRI.

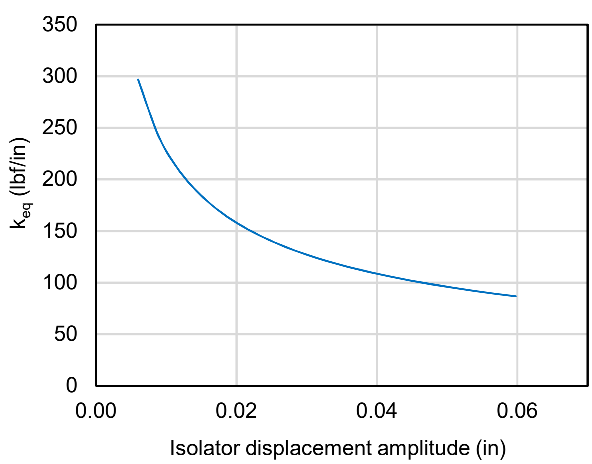

Figure 8. k_{eq} vs. isolator displacement amplitude for a typical WRI

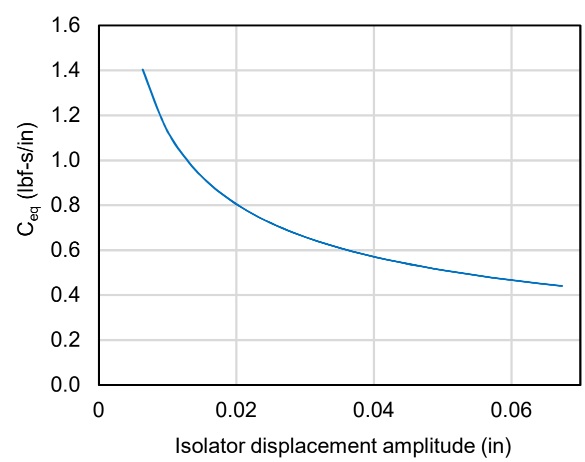

Figure 9. C_{eq} vs. isolator displacement amplitude for a typical WRI

The Socitec Group established these equivalence curves for each WRI and elastomeric model through repeated testing and the use of similitude.

It is interesting to note that a WRI vibrating at small amplitude experiences no Coulomb damping, as there is no friction developed due to the lack of relative displacement between the wires and the strands. In this region, the stiffness reaches the so-called limit stiffness, and the equivalent damping tends to zero. When cycled in this region, the WRI behaves as an undamped spring, providing the best possible isolation. This is usually the case for higher frequency vibration.

Additionally, Figure 9 shows that the equivalent viscous damping ratio diminishes at very large vibration amplitudes. This is because at very large vibration amplitudes the energy removed from the system by Coulomb damping is small compared to the energy stored by deflection of the WRI. Finally, it is important to note that a primary difference between elastomeric and WRIs is that the dynamic stiffness of a WRI depends only on its displacement amplitude, while the dynamic stiffness of an elastomeric isolator is dependent on both the amplitude and frequency of vibration.

3. Shock theory

Unlike steady-state vibration, shock is transient by nature and can be modeled as either an instantaneous change in velocity or displacement. A container colliding with the support from a drop height h, or a sprung mass experiencing shock subsequent to a non-contact underwater (NCU) explosion, are two examples of how a system could be exposed to shock.

A shock isolator's purpose is to reduce the severity of the shock experienced by the sprung mass. The isolator accomplishes this by storing the shock energy and then releasing it over a longer period, in a form that is less likely to cause damage to critical elements within the sprung mass. When both shock and vibration are to be accounted for in choosing an isolator, shock is usually the sizing factor.

Shock input is often defined as a drop impact or by an impulse, for example: half-sine, triangular, and rectangular waveforms. Therefore, a system experiencing a shock first responds to the input, then oscillates freely before coming back to rest.

3.1 Linear systems

3.1.1 *Free vibration*

A system undergoes free vibration if it is given initial conditions (initial displacement or velocity), then allowed to vibrate freely. This is equivalent to solving the ordinary differential equation of the mass-spring-damper with $u(t) = \dot{u}(t) = 0$. The solution to this differential equation is:

$$x(t) = x_0 * e^{-\xi\omega_n t}\sin(\omega_d t - \varphi)$$

Where x_0 (amplitude) and φ (phase shift) both depend on the initial conditions given to the system.

An important point to take away from this result is that if the system is allowed to freely vibrate, then it will vibrate at its damped natural frequency ($\omega_d = \omega_n\sqrt{1-\xi^2}$) before the vibration decays to zero. Therefore a system experiencing a shock will always vibrate freely once the input disappears. An example of a free vibration response is shown in Figure 10.

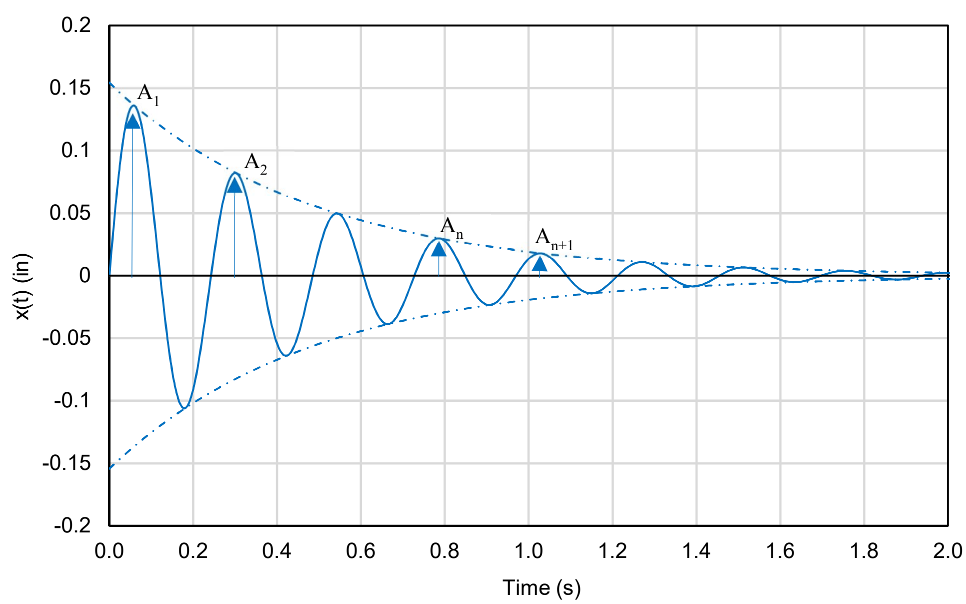

Figure 10. Post-shock free vibration of a typical underdamped, linear mass-spring-damper system

2443 Braga Drive, Broadview, Illinois 60155-3941 USA 708-345-2050 1-800-842-7668 Fax 708-345-2225 www.vibrodynamics.com vibro@vibrodynamics.com

A useful quantity in free vibration is the logarithmic decrement. The logarithmic decrement is the rate at which motion of a system decays. Mathematically, it is defined as:

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} = \ln\left(\frac{A_n}{A_{n+1}}\right)$$

In free vibration, the higher the damping in a system the faster the vibration decays to zero. For example, WRIs with about 17% damping ratio will decay in less than three oscillations. Chloroprene rubber, with a damping ratio of around 10%, will decay in less than six oscillations.

3.1.2 Shock response spectrum (SRS)

Explained above is how a linear system responds beyond the shock input. The SRS is a useful tool in determining how a linear system responds initially during a shock. The shock response concept provides a powerful yet simple way to characterize a signal. If a body M experiences any transient signal as part of a global system (e.g. Figure 10), it is possible to assess the worst case response of its critical components by means of its SRS. Basically, if the mass of the global system M is very large compared to the critical components' masses m_i, then the response of the body M can be considered as the input to the masses, m_i. For example, if Figure 10 is the time history response of the body M, it will then be used as the input to the critical masses, m_i. These critical masses, m_i, are idealized as massless oscillators, and tuned at any desired range of natural frequencies, for instance 1–1000 Hz. This model is shown in Figure 11. It is important to understand that this spectrum can be calculated at any location of the system to then evaluate the response of the critical components at that location in the corresponding axis of motion. If the SRS of the time history input is used, then it is often called the design SRS. The design SRS gives the response of different linear isolators across a range of natural frequencies. It is called the design SRS because it is used to compare how different isolators with different natural frequencies would respond to a given input signal. The design SRS is mostly used for naval and seismic applications to provide a global overview of the system's response.

One other extremely useful function of the SRS is its use for defining a qualification test. The advantage of this is that none of the critical components' natural frequencies need to be known, because the provided qualification input will generate a response that is more severe than the one seen in practice across all frequencies (e.g. 1–1000 Hz). The process of defining a qualification test is summarized below.

First, the system is modeled with the chosen WRIs, and the time history response is determined. This can be done using Socitec Group's simulation software, SYMOS. The SRS of this time history response is then calculated to assess the response of the system's critical components. An equivalent input is then defined, which would envelope the previously calculated SRS, an example is shown in Figure 12. This input is in a form compatible with lab machines (e.g. ½ sine pulse) for testing the hard mounted system without the WRIs. If the system and all its critical components survive the test, then the chosen WRIs will protect these components from the actual shock that will be experienced in the field. This process serves as a qualification test before having the WRIs in hand. Vibro/Dynamics can provide this service upon request.

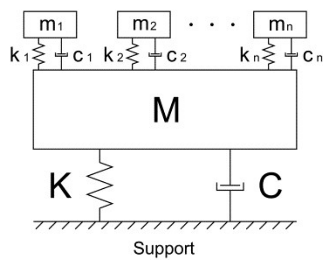

Figure 11. Model of global system, M, and its n massless critical components

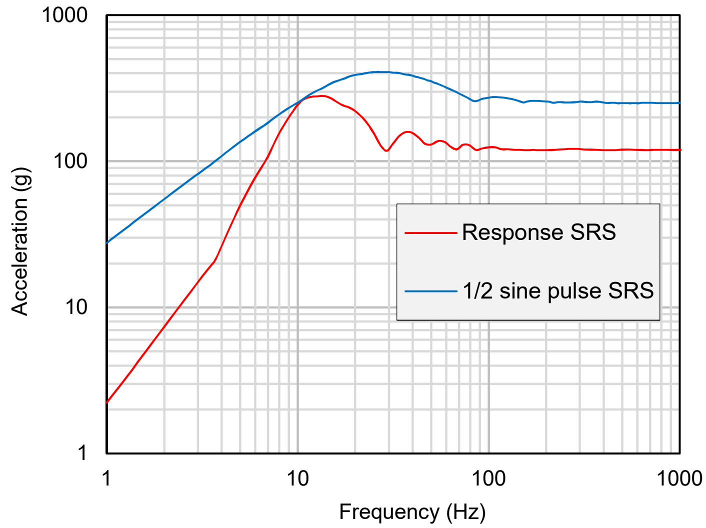

Figure 12. Example representation of the equivalent pulse input compared with the response SRS of the system

3.1.3 *Linear energy equating method*

In some cases shock input is specified as an acceleration pulse, which can be characterized by its velocity change Δv if the duration of the pulse input is short compared to the response's duration. This velocity change is determined by the following formula:

$$\Delta v = \int_0^\tau a(t)dt = \frac{2a_0\tau}{\pi}$$

$a(t)$: time history of the acceleration input, a_0: amplitude of the acceleration pulse, τ: duration of the acceleration pulse.

Using this velocity change Δv, the shock energy of the system with mass m can be found by its kinetic energy:

$$KE = \frac{1}{2}m\Delta v^2$$

This kinetic energy is then equated to the potential energy in the isolators, when they are deflected to a maximum. In this case four identical isolators are assumed:

$$PE = 4\left(\frac{1}{2}k_{shoc}\ {x_{max}}^2\right)$$

x_{max}: maximum dynamic displacement of each isolator, $k_{shoc}\ = F_{shock}/d_{shock}$: average shock stiffness of each isolator.

The average shock stiffness of an isolator is directly determined from its corresponding datasheet with the chosen attitude (e.g. Compression and Tension, Shear or Roll). The different mounting attitudes are illustrated in page 21.

Equating kinetic and potential energy:

$$4\left(\frac{1}{2}k_{shock}x_{max}^2\right) = \frac{1}{2}m\Delta v^2$$

Then:

$$x_{max} = \sqrt{\frac{\Delta v^2 m}{4k_{shock}}}$$

Using Newton 2nd Law: $\sum F = ma$:

$$4k_{shock}x_{max} = ma_{max}$$

$$a_{max} = \frac{4k_{shock}x_{max}}{m}$$

Shown above are the steps used to find the maximum dynamic displacement of an isolator and the corresponding maximum acceleration of the sprung mass. This calculation applies only for a linear, 1-DOF system rarely seen in practice. Depending on the application sensitivity, this linear approximation method might be enough to yield the solution. However, for more sensitive applications it is strongly recommended to use nonlinear multiple degrees of freedom methods to find the solution.

3.2 Nonlinear systems

Shock is also strongly influenced by nonlinearity, however applying the same linearization procedure as discussed in vibration nonlinearity does not hold for shock problems. Due to the high nonlinearity of WRIs, especially when involving large displacements seen in shock applications, the response is best evaluated with real load deflection curves, as shown in Figure 13.

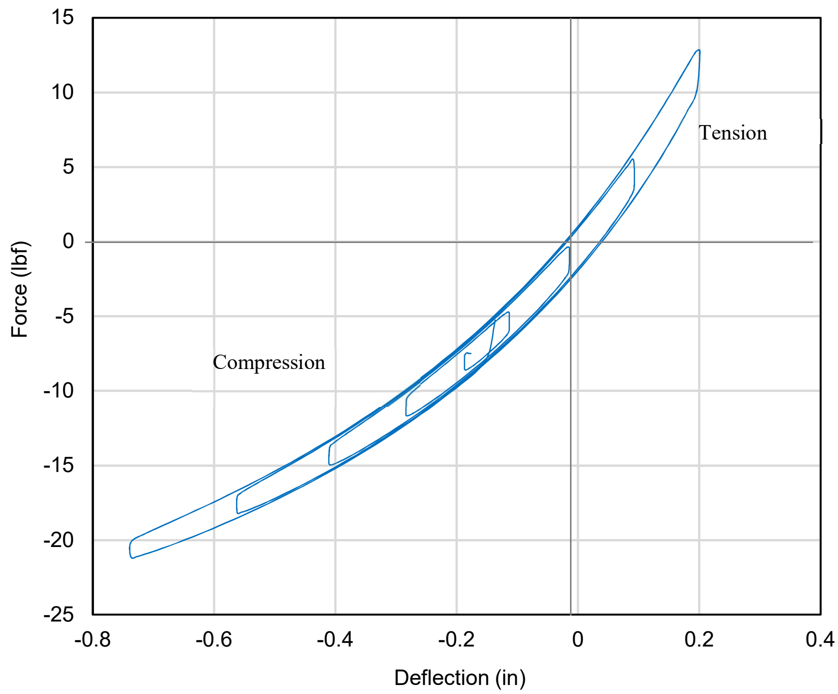

Figure 13. Load-deflection curve of a typical WRI in compression and in tension

Determining a solution for nonlinear shock problems is done in two different ways, depending on the input:

- Nonlinear energy equating method
- Direct integration of the differential equations

3.2.1 *Nonlinear energy equating method*

As seen in section 3.1.3, if the duration of the pulse input is short compared to the duration of the response, then a shock input can be characterized by its velocity change, Δv. Using this velocity change, the shock energy can be found by its kinetic energy $1/2m\Delta v^2$, which is equated to the potential energy in the nonlinear isolators, when they are deflected to a maximum. There are two methods to determine the response amplitude in a nonlinear isolator:

1. The simpler method is to use the linear equation for potential energy in an isolator, and multiply it by an adjustment factor, $C_{nonlinear}$, to account for nonlinearity. The potential energy in the four identical nonlinear isolators becomes:

$$PE_{nonlinear} = C_{nonlinear}\left(4\frac{1}{2}k_{shock}x_{max}^2\right)$$

The adjustment factor, $C_{nonlinear}$, varies depending on the isolator's mounting attitude. The different mounting attitudes are illustrated in page 21. For typical shock deflections seen in practice, refer to Table 1 for approximate $C_{nonlinear}$ values.

Mounting attitude	Compression	Tension	Shear or Roll
$C_{nonlinear}$	1.3	0.7	0.7

Table 1. Adjustment factor per mounting attitude for typical shock deflections

2. A more elaborate method to find the response amplitude of a nonlinear isolator is to use its load deflection curve. In this method, the kinetic energy of the system is equated to the area under the load deflection curve of the isolator. The corresponding response amplitude (x_{max} and a_{max}) is then readily determined. Vibro/Dynamics can provide this calculation, or provide the load deflection curve of the isolator of interest.

3.2.2 *Direct integration of differential equations*

The method explained previously for treating nonlinear systems applies when the input duration is short compared to the duration of the response. However, when this is not true the shock input can no longer be characterized by its velocity change, Δv. The energy equating method cannot be applied for such inputs. Finding the right solution is then best carried out through direct integration of the differential equations, using software programs such as Socitec Group's SYMOS.

4. Multiple degrees of freedom systems

The theories and practices previously discussed only apply to 1-DOF systems. The selection of isolators becomes further complicated when analysis requires the consideration of additional degrees of freedom. These could be due to rotations, flexibility of the equipment or supporting structures, or the presence of multiple bodies in the system. In some cases, it is acceptable to solve these additional degrees of freedom independently, simplifying the solution process. However, there are many cases where multiple degrees of freedom are coupled, requiring a more sophisticated model to obtain the global dynamic behavior of the system. The same differential equation of motion is used, but with some modifications. The variables and constants are replaced by matrices, and the solution is completed through the use of eigenvalues and eigenvectors. Above three degrees of freedom, it is impractical to carry out such calculations by hand, necessitating the use of nonlinear simulation programs, such as Socitec Group's SYMOS or finite element programs that can handle hundreds of degrees of freedom.